MAT305 Exam 1

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
This exam has 2 questions for a total of 25 points.

Name and section: _____

Instructor's name: _____

Question:	1	2	Total
Points:	10	15	25
Score:			

1. [10 points] Solve the separable differential equation

$$y' + t^3 y^2 = t^3.$$

Solution: We begin by manipulating:

$$y' = t^3(1 - y^2)$$

There are the constant solutions $y = \pm 1$. Discarding the possibility that $1 - y^2 = 0$ (on the strength of the uniqueness theorem for linear DEs), we have

$$\begin{aligned}
\frac{y'}{1-y^2} &= t^3 \\
\left(\frac{1}{1-y} + \frac{1}{1+y}\right) y' &= 2t^3 \\
\frac{d}{dt}(\ln|1+y| - \ln|1-y|) &= 2t^3 \\
\ln\left|\frac{1+y}{1-y}\right| &= \frac{t^4}{2} + C \qquad (C \text{ arbitrary}) \\
\frac{1+y}{1-y} &= ce^{t^4/2} \qquad (c \text{ arbitrary}) \\
y &= \frac{ce^{t^4/2} - 1}{ce^{t^4/2} + 1}
\end{aligned}$$

Answer: _____

2. Solve the differential equation

$$y' + 14y = t$$

using an integration factor, using the following procedure.

(a) [5 points] What is the integrating factor $\mu(t)$?

(a) _____

(b) [5 points] What function is $\mu(t)y' + 14\mu(t)y$ the derivative of?

(b) _____

(c) [5 points] Find the general antiderivative of both sides of $\mu(t)(y' + 14y) = \mu(t)t$ and solve for y.

Solution: The integrating factor $\mu(t)$ should satisfy $\mu'(t) = 14\mu(t)$, so we choose

$$\mu(t) = e^{14t}.$$

This makes the left side the derivative of the expression

$$e^{14t}y$$

according to the product rule. Therefore, by taking the general antiderivative of the indicated expression, we have

$$\begin{aligned} e^{14t}y &= \int te^{14t}\,dt \\ &= \frac{1}{14}te^{14t} - \frac{1}{196}e^{14t} + C \qquad (C \text{ arbitrary}) \\ y &= \frac{t}{14} - \frac{1}{196} + Ce^{-14t}. \end{aligned}$$